Calculus Warm Up #1-3

$x^2 + y^2 = r^2$ $\quad \tan\theta = \dfrac{y}{x}$

$$\boxed{\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \end{aligned}}$$

Rewrite the polar equation in rectangular form.

1. $r = 4\sin\theta$

2. $r = \dfrac{6}{2 - 3\sin\theta}$

Polar WS Answers:

1) $(0, 4)$
2) $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
3) $(2, -2\sqrt{3})$
4) $(0, 0)$
5) $\left(\sqrt{2}, \frac{\pi}{4}\right)$
   $\left(-\sqrt{2}, \frac{5\pi}{4}\right)$
6) $\left(5, \frac{3\pi}{2}\right)$
   $\left(-5, \frac{\pi}{2}\right)$
7) $\left(\sqrt{6}, \frac{5\pi}{4}\right)$ & $\left(-\sqrt{6}, \frac{\pi}{4}\right)$
8) $r = 3$
9) $r = 2\cos\theta$
10) $r = 4\csc\theta$
11) $r = -\dfrac{2}{3\cos\theta - \sin\theta}$
12) $r = \dfrac{2}{4\cos\theta + 7\sin\theta}$
13) $\theta = \frac{\pi}{4} + \pi n$
14) $r = \dfrac{4}{1 - \cos\theta}$ or $\dfrac{-4}{1 + \cos\theta}$
15) $y = \frac{\sqrt{3}}{3}x$

HW Questions:

14) $y^2 - 8x - 16 = 0$

$(r\sin\theta)^2 - 8(r\cos\theta) - 16 = 0$

$r^2\sin^2\theta - 8(r\cos\theta) - 16 = 0$

$r^2(1-\cos^2\theta) - 8(r\cos\theta) - 16 = 0$

$r^2 - (r\cos\theta)^2 - 8(r\cos\theta) - 16 = 0$

$r^2 = (r\cos\theta)^2 + 8(r\cos\theta) + 16$

$1 \cdot 16$
$2 \cdot 8$
$4 \cdot 4$

$\sqrt{r^2} = \pm\sqrt{(r\cos\theta + 4)^2}$

$r = r\cos\theta + 4 \qquad r = -r\cos\theta - 4$

Finding slopes and points of tangency for polar curves.

$\frac{dy}{dx} \qquad (r, \theta)$

where $r = f(\theta)$

Example polar curve:

$r = 2 - 2\cos\theta$

$f(\theta) = 2 - 2\cos\theta$

$f'(\theta) =$

In General

If r is a differentiable function of θ, $r = f(\theta)$, then the slope of the tangent is:

$f(\theta) \cdot \sin\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \qquad \begin{array}{l} (y = r\sin\theta) \longrightarrow \dfrac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta} \\ (x = r\cos\theta) \longrightarrow \end{array}$$

$f(\theta)$

Ex: polar curve: $f(\theta)=r$ $\quad f'(\theta)=2\sin\theta$

r = 2 - 2cosθ

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

$$\frac{dy}{dx} = \frac{2(1-\cos\theta)\cos\theta + 2\sin\theta\sin\theta}{-2(1-\cos\theta)\sin\theta + 2\sin\theta\cos\theta}$$

$$= \frac{\cancel{2}(\cos\theta - \cos^2\theta + \sin^2\theta)}{-\cancel{2}\sin\theta(1-\cos\theta-\cos\theta)} \qquad \begin{array}{l}\sin^2\theta \\ 1-\cos^2\theta\end{array}$$

$$= +\frac{(+2\cos^2\theta - \cos\theta - 1)}{+\sin\theta(1-2\cos\theta)}$$

$$= \frac{(2\cos\theta+1)(\cos\theta-1)}{\sin\theta(1-2\cos\theta)}$$

Ex: polar curve: $f(\theta)=r$ $\quad f'(\theta)=2\sin\theta$

r = 2 - 2cosθ

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

$$\frac{dy}{dx} = \frac{(2\cos\theta+1)(\cos\theta-1)}{\sin\theta(1-2\cos\theta)}$$

Horiz tangent

$2\cos\theta + 1 = 0 \qquad \cos\theta - 1 = 0$

$\cos\theta = -\frac{1}{2} \qquad \cos\theta = 1$

$\theta = 0$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

$$\boxed{\left(3, \tfrac{2\pi}{3}\right) \;\&\; \left(3, \tfrac{4\pi}{3}\right)}$$

r ?

$r = 2(1-\cos\theta)$

Vert

$\sin\theta = 0 \qquad 1-2\cos\theta = 0$

$\theta = 0, \pi \qquad \cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

$(4, \pi) \quad \left(1, \frac{\pi}{3}\right) \quad \left(1, \frac{5\pi}{3}\right)$

You try:

Find $\frac{dy}{dx}$ and slopes of the tangents at (r, θ)

where $r = f(\theta)$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

$r = 2 + 3\sin\theta$ at: $(2, \pi)$ and $(5, \frac{\pi}{2})$

$f'(\theta) = 3\cos\theta$

Slope $= 0$

$$\frac{dy}{dx} = \frac{2\cos\theta(1 + 3\sin\theta)}{-(6\sin^2\theta + 2\sin\theta + 3)}$$

@ $(2, \pi)$ slope $= -\frac{2}{3}$

HW:

p. 707 # 3 - 9, skip 7